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A STUDY ON SOLID VIBRATIONS IN ONE-STORY BUILDINGS OF SPECIAL SHAPES

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A STUDY ON SOLID VIBRATIONS IN ONE-STORY BUILDINGS OF SPECIAL SHAPES

(L-SHAPED and]-SHAPED)

By

Dr MUTO Kiyoshi - Engineering Department of Tokyo Imperial University

and

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Study Relating to Solid Vibrations in One-Story Buildings of Special Shapes

(L-Shaped and コ-Shaped)

Dr MUTO Kiyoshi

ASAGA Hirozumi

Summary of Contents

This thesis deals with solid vibrations in L shaped and コ shaped buildings of one story. These types of structures, even when the mass is evenly distributed are easily subjected to stresses and will consequently vibrate. We shall indicate the factors governing this solid vibration and, taking L and コ shapes with arms of various lengths, we will study their individual solid vibrations.

Table of Contents

1. Foreword

Buildings constructed on a square plan have uniform longitudinal and lateral rigidity. The central point C of rigidity is at the geometrical center. When the mass is evenly distributed there is no solid vibration as the center of gravity S corresponds with the center of rigidity C. However, in plane figures having unequal longitudinal and latitudinal axes, but having uniform rigidity and even distribution of mass, the center of rigidity and the center of gravity do not correspond and a purely lateral vibration or a torsional vibration does not occur. In general, however, it may reasonably be presumed that solid vibration depends as a rule on the distribution of mass.

Bearing these facts in mind, therefore, we shall take these special L-shaped and コ-shaped Figures and study the nature of their solid vibrations, namely of one story L - and コ-shaped Frame structures with equal distribution of framework and uniform rigidity of lateral and longitudinal framework.

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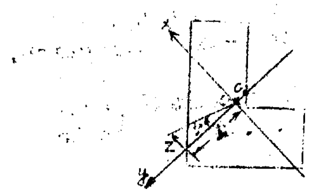
Symbols

The agreed symbols are as follows:

Displacement: $x, y, \varphi = \frac{z}{i}$



The displacement of the center of gravity S is x and y. φ stands for the angle of rotation of the circumference. The direction as shown in Figure 1 is taken as positive.

Figure 1. Symbols

Mass: M
Polar Moment of Inertia: $J = M \cdot i^2$
Radius of Rotation: i

The radius of rotation of the circumference of center of gravity S is i.

Characteristic Coefficients: C (Restoring Force Direction) / (Displacement Direction)

It is the restoring force of gravity per unit displacement in a certain direction. The lower row indicates the displacement direction and the upper row indicates the restoring force.

Equations of Vibration

In this study, in dealing with rigidity in Longitudinal and lateral directions and taking them as equal, as already described, the principal axis may be any axis passing through the central point of rigidity. Therefore considering the y axis to run in an eccentric direction. The vibration on the y -axis direction will be free vibration. The joint vibration in the directions of the x and z axes will be the so-called solid vibration. Their vibration formulae are as follows:-

$$\left.\begin{aligned} C_x^{(x)} \cdot x + C_z^{(x)} \cdot z &= -M\ddot{x} \\ C_x^{(z)} \cdot x + C_z^{(z)} \cdot z &= -M\ddot{z} \end{aligned}\right\} \text{------------- (1)}$$

Thus, $C^{(x)}_{x}$: The gravitational restoring force in the x - axis direction per free unit displacement in the x - axis direction.

$C^{(z)}_{x} = C^{(x)}_{z}$: The gravitational restoring force in the z-axis or x-axis direction per free unit displacement in the x-axis or z-axis direction.

$C^{(z)}_{z}$: The restoring force in the x-axis direction per free unit displacement (i.e. $\phi = \frac{1}{i}$) in the z-axis direction.

This resolves into: $x = u \cdot q$
$x = w \cdot q$

Where u,w: characteristic function

q : time function

And $q = n^2 q$
$q = A \cos nt - B \sin nt$
n : circular frequency (angular velocity)

The characteristic function equation may be expressed as

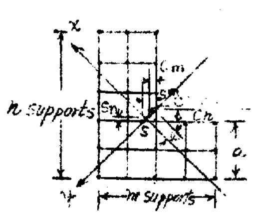
$$\left.\begin{aligned} (C^{(x)}_{x} - Mn^2) \cdot u - C^{(x)}_{z} \cdot w &= 0 \\ C^{(z)}_{x} \cdot u - (C^{(z)}_{z} - Mn^2)\, w &= 0 \end{aligned}\right\} \text{------------------} \quad (2)$$

The circular frequency n may be determined from the following root equation, which is the determinant of the coefficients of the above equation set equal to zero.

$$\begin{vmatrix} (C^{(x)}_{x} - Mn^2) & C^{(x)}_{z} \\ C^{(z)}_{x} & (C^{(z)}_{z} - Mn^2) \end{vmatrix} = 0 \text{ ---------------------------} \quad (3)$$

2. Solid Vibration in an L-Shaped Frame Centers of Rigidity and Gravity

As shown in Figure 2 the width of the L-shaped structure is a. The number of supports is as follows:-



Across the width - 3 (three)

Longitudinally - n

Laterally - m

Figure 2. Symbols for Structure of L-Shaped Frame

Using the symbols defined in the figure, the position of the center C of rigidity may be expressed as follows:-

$$Cm = \frac{(m-3)(m-2) - 2n}{4(m+n-3)} \cdot a$$

$$Cn = \frac{(n-3)(n-2) - 2m}{4(m+n-4)} \cdot a \qquad \text{---------------} \quad (4)$$

And the position of the center of gravity S is shown as follows:-

$$S_m = \frac{(m-3)^2 - 2(n-1)}{4(m+n-4)} \cdot a$$

$$S_n = \frac{(n-3)^2 - 2(m-1)}{4(m+n-4)} \cdot a \qquad \text{---------------} \quad (5)$$

Hence, taking the longitudinal rigidity and lateral rigidity as equal, and with this limitation, this becomes

$$Cm - Sm = Cn - Sn = \frac{mn - 3(m+n) + 9}{4(m+n-3)(m+n-4)} \cdot a$$

As shown in this formula, the line joining the center of gravity with the center of rigidity, i.e., in an eccentric direction, must form an angle of 45° with the rows of supports, regardless of the values of m and n.

Characteristics Coefficients

If rigidity is expressed as G (restoring force of the center of gravity in the x-axis direction per unit displacement in the direction of the x-axis, that is, rigidity in the x-direction) then by definition

$$C^{(x)}_{x} = G \qquad \text{---------------} \quad (6)$$

and hence $G = 3(m + n - 3)g$

g being the rigidity of one support.

Furthermore, from the turning moment of the restoring force center of gravity in the x-axis direction is derived:

$$C^{(z)}_{x} = C^{(x)}_{z} = -Ge' \qquad \text{---------------} \quad (7)$$

But $\quad e' = \frac{e}{i}$

then, if one assumes an angle of rotation such that $z = i\varphi = 1$

on the rotation of the center of gravity, the center of rigidity suffers the following resultant displacements:

$x = -e'$, in the direction of the x-axis
$z = 1$, in the direction of the z-axis

We seek separately the moments given to S by the two components so that $C^{(z)}_z$ is obtained as follows:-

$$C^{(z)}_z = c \cdot G \quad \text{-----------------} \quad (8)$$

Where
$$c = \frac{1}{3(m+n-3)\,i^2}\left\{\sum m'\eta^2 + \sum n'\xi^2\right\} + e'^2$$

and where
$$\sum_{\eta} m'\eta^2 = \frac{(n-3)(n-2)(2n-5)}{8}\,a^2 + \frac{Sm}{4}\,a^2 - 3(m+n-3)C_n^2$$

$$\sum_{\xi} n'\xi^2 = \frac{(m-3)(m-2)(2m-5)}{8}\,a^2 + \frac{5n}{4}\,a^2 - 3(m+n-3)C_m^2$$

But m' also stands for m or 3, and n' for n or 3.

Characteristic Circular Frequency and Vibration Curve

The characteristic coefficients having been established as shown above, we now take equation (2) and rewrite it as follows:

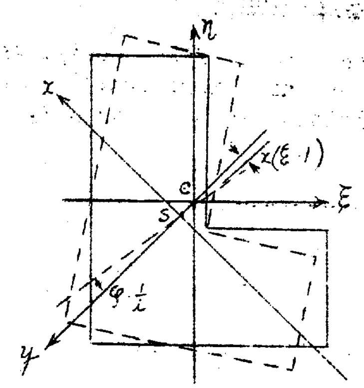


Figure 3. Displacement of the central point of rigidity accompanying the rotation of the circumference of the center of gravity.

$$\left.\begin{aligned}\left\{1-\left(\frac{n}{n_o}\right)^2\right\} u - e'w &= 0\\ -e'u + \left\{e-\left(\frac{n}{n_o}\right)^2\right\} w &= 0\end{aligned}\right\} \text{--} \quad (9)$$

where $n_o = \sqrt{\frac{G}{M}}$, that is,

n_o stands for the circular frequency when there is free vibration in the direction of the x-axis.

By setting the determinant of this equation equal to zero, the circular frequency may be expressed as follows:-

$$n^2 = \tfrac{1}{2}\left\{(c+1) \pm \sqrt{(c-1)^2 + 4e'^2}\right\} n_o^2 \quad \text{-------} \quad (10)$$

Accordingly, taking u = 1 in equation (9), the characteristic function, the following equation is obtained:-

$$w = \frac{\left(1 - \frac{n}{n_o}\right)^2}{e'} \quad \text{---------------} \quad (11)$$

Thus u = 1

Vibration Characteristics

1. As is evident in equation (10), since n contains - and - it has two values. We shall distinguish between these two by writing them n_x and n_z depending on whether the vibration is lateral or rotary.

Thus, from the vibration equation (10), and multiplying n_x and n_z, its properties are seen to be as follows:-

$$n_x \cdot n_z = \sqrt{c - e'^2} \cdot n_o^2$$

$$= \sqrt{\frac{1}{3(m+n-3)i^2}} \left\{\sum m'\eta^2 + \sum n'\xi^2\right\} \cdot n_o^2$$

$$\fallingdotseq \text{an expression independent of } e' \quad \text{----} \quad (12)$$

2. In equation (9) u is set equal to 1 and $(n/n_o)^2$ is cancelled out; therefore a quadratic equation in w may be expressed thus -

$$w^2 + \frac{c-1}{e'}w - 1 = 0$$

From this, two roots for w are obtained.

The following relation holds between them, as with w_x and w_z from (11) in its unaltered form:-

$$w_x \cdot w_z = 1 \quad \text{----------} \quad (13)$$

3. Furthermore, lateral vibration and rotational vibration have as center of rotation: Dx, Dz.

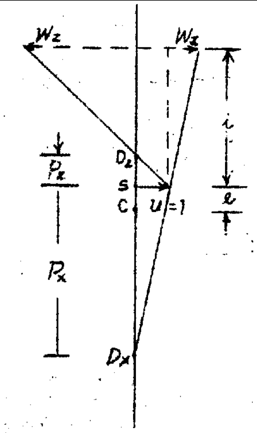


Figure 4. Distance of Centers of Rotation

Hence, as shown in Figure 4, their distances from the center of gravity are Px and Pz respectively:

$$P_x = \frac{i}{w_x}, \quad P_z = \frac{i}{w_z}$$

From this, and applying equation (13), we get the relation:

$$P_x \cdot P_z = -i^2 \quad (14)$$

The above three properties of vibration always exist in one-story frame structures and should all be taken into consideration.

3. <u>An Example of Solid Vibration in an L Shaped Frame</u>

We shall now take five different types of L shaped frame structures, designated A, B, C, D and E, with a number of lateral and longitudinal supports, and shall study their bodily vibration. In these types of structures the number of supports is as shown in Table 1. This table also gives the area and the ratio λ between the longitudinal length and the lateral length of each structure.

Table 1.

Type	m	n	λ	Area
A	7	7	1	5 a^2
B	9	9	1	7 a^2
C	7	11	0.6	7 a^2
D	9	15	0.6	10 a^2
E	7	13	0.5	8 a^2

In the following table values are given respectively for: i, the radius of rotation of the center of gravity S; e, the eccentric distance obtained by calculations based on equations (4) and (5); $e' = \frac{e}{i}$; and lastly for the coefficient c as shown in equation (8), namely the ratio between rotational rigidity and lateral rigidity.

Table 2. Characteristic Coefficients for Various Types of L-Shaped Structures

Type	m:n	i	e	e'	c
A	7:7	1.203 a	0.051 a	0.042	1.2848
B	9:9	1.642 a	0.061 a	0.037	1.1932
C	7:11	1.715 a	0.054 a	0.031	1.1789
D	9:15	2.428 a	0.068 a	0.028	1.1166
E	7:13	2.003 a	0.052 a	0.026	1.1453

Having established these values, we can now obtain circular frequencies, characteristic functions, and the distance from the center of rotation, as given in the following table:

Table 3. Nature of the Vibration of Various Types Types of L-Shaped Structures

Type	m:n	n_x/n_o	n_z/n_o	w_x	w_z	P_x/a	P_z/a
A	7:7	0.9970	1.1361	0.1444	-6.9242	8.3299	-0.1737
B	9:9	0.9966	1.0955	0.1850	-5.4062	8.8770	-0.3037
C	7:11	0.9974	1.0882	0.1684	-5.9379	10.1835	-0.2888
D	9:15	0.9968	1.0597	0.2276	-4.3936	10.6674	-0.5526
E	7:13	0.9977	1.0723	0.1735	-5.7623	11.5420	-0.3476

The deductions from these results are as follows: The changes in vibration and characteristic functions created by the shape of the figure are almost negligible.

When there is lateral vibration, the distance of vibration from the center of rotation varies in direct ratio to the length of the arms of the figure. That is to say, when the arms of the figure are short the degree of eccentricity e' is diminished, so that the distance from the center of rotation tends to be greater. (Rotation during lateral vibration is slight.)

Each type of vibration is shown in diagram form in Figures 5 to 9. In these figures the heavy lines show the structures in position of rest or equilibrium; the fine lines represent lateral vibration and the dotted lines circular vibration.

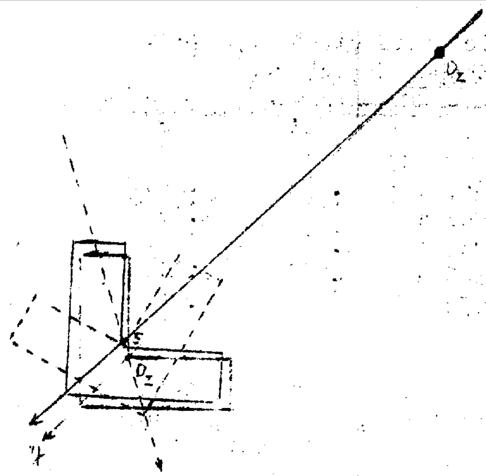


Figure 5. The form of vibration in Type A (m:n = 7:7)
$P_x = 8.3299$ a, $P_z = -0.1737$ a

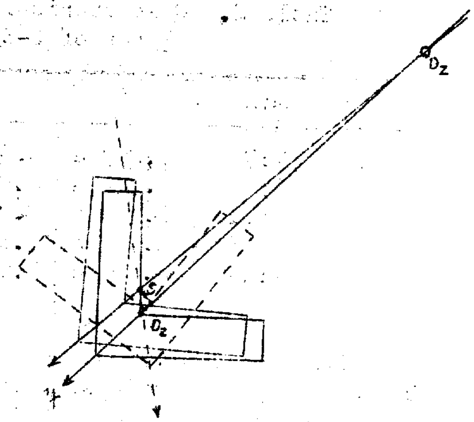


Figure 6. The form of vibration in Type B (m:n = 9:9)
$P_x = 8.8770$ a, $P_z = -0.3037$ a

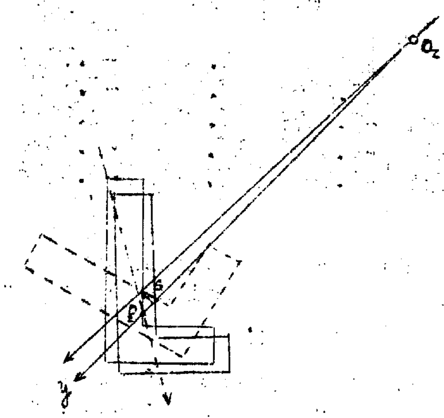


Figure 7. The form of vibration in Type C (m:n = 7:11)
$P_x = 10.1835$ a,
$P_z = -0.2888$ a

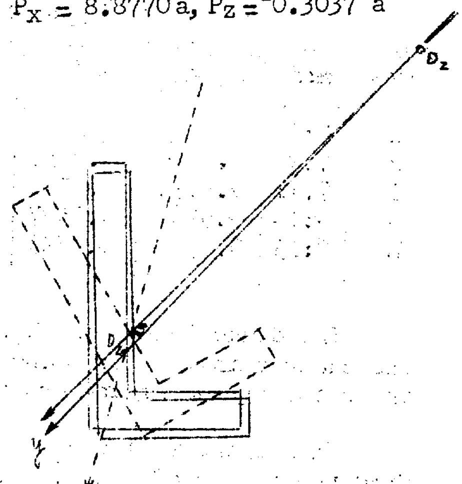


Figure 8. The form of Vibration in Type D (m:n = 9:15)
$P_x = 10.6674$ a, $P_z = -0.5526$ a

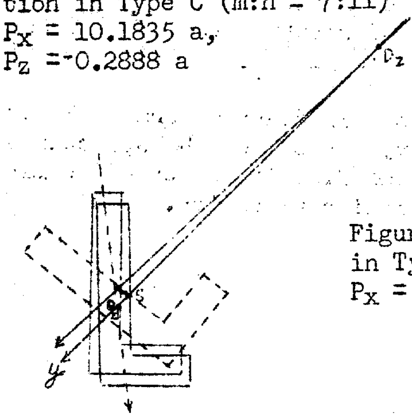


Figure 9. The form of vibration in Type E (m:n = 7:13)
$P_x = 11.5420$ a, $P_z = -0.3476$ a

4. Solid Vibration in a] -Shaped Frame

The Center of Rigidity and the Center of Gravity

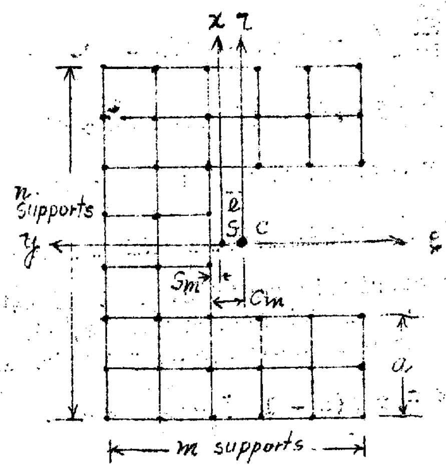


Figure 10. Symbols for] -Shaped Structures

As shown in Figure 10, in] -shaped structures "a" stands for width, 3 is the number of supports across the width and n and m are the number of supports longitudinally and laterally respectively. When the axes are symmetrical the center C of rigidity is located as follows:-

$$C_m = \frac{(m - 3)(m - 2) - n}{2(2m + n - 6)} a \quad - - - - - - - - - - \quad (15)$$

and the position of the center of gravity when the axes are symmetrical can be determined as follows:-

$$S_m = \frac{(m - 3)^2 - (n - 1)}{2(2m + n - 7)} a \quad - - - - - - - - - \quad (16)$$

Characteristic Coefficients

The characteristic coefficients for] -shaped structures are the same as those for L-shaped structures and may be expressed as follows:-

$$C_x^{(x)} = G$$

$$C_x^{(z)} = C_z^{(x)} = -Ge'$$

$$C_z^{(z)} = \frac{e}{i}$$

Thus $e' = \frac{e}{i}$

$$C = \frac{1}{3(2m + n - 6)i^2}\left\{\sum_{\eta} m'\eta^2 + \sum_{\xi} n'\xi^2 + e'^2\right.$$

$$\sum_{\eta} m'\eta^2 = \frac{a^2}{16}\left[(n-5)(n-6)(n-7) + 2m\left\{(n-5)^2 - (n-3)^2 - (n-1)^2\right\}\right]$$

$$\sum_{\eta} n\xi^2 = \frac{a^2}{4}\left\{5n + (m-3)(m-2)(2m-5) - 3(2m + n - 6)C_m^2\right.$$

But m' stands for m or 3, and n' for n or 3.

Characteristic Values of Vibration and Vibration Curve

To fix characteristic coefficients, we take the same formulas for characteristic vibration values, characteristic function and distance from rotation center as for L-shaped structures.

Thus:- $$n^2 = \frac{1}{2}\left\{(c + 1) \pm \sqrt{(c - 1)^2 - 4e'^2}\right\}\cdot n_0^2$$

$$w = \frac{1 - \left(\frac{n}{n_0}\right)^2}{e'}$$

$$p = \frac{i}{w}$$

5. Examples of Solid Vibration in a ℶ -Shaped Frame

We will now investigate the vibration characteristics of four types of ℶ -shaped frame structures, designated F, G, H and I, having the number of supports, area and ratio of the lengths of the sides as specified in the following table:

Table 4

Type	m	n	λ	Area
F	7	7	1	7 a^2
G	9	9	1	10 a^2
H	7	9	0.75	8 a^2
I	7	13	0.5	10 a^2

In the following table values are given respectively for: i, the radius of rotation of the center of gravity S; e the eccentric distance calculated from equations (15) and (16); e' the degree of eccentricity; and c, the ratio between rotational rigidity and lateral rigidity.

Table 5. Characteristic Coefficient for each Type of ℶ -Shaped Structure

Type	m:n	i	e	e'	c
F	7:7	1.311 a	0.076 a	0.058	1.2226
G	9:9	1.838 a	0.093 a	0.051	1.1310
H	7:9	1.612 a	0.074 a	0.046	1.1635
I	7:13	2.249 a	0.067 a	0.030	1.0993

Having established these values, we can now obtain the values of vibration, characteristic functions, and distance from the center of rotation, as given in the following table:

Table 6. Nature of the Vibration of Various Types of ⊐ -Shaped Structures

Type	m:n	n_x/n_o	n_z/n_o	w_x	w_z	P_x/a	P_z/a
F	7:7	0.9929	1.1121	0.2449	-4.0831	5.3530	-0.3211
G	9:9	0.9912	1.0717	0.3435	-2.9113	5.3508	-0.6314
H	7:9	0.9940	1.0842	0.2620	-3.8162	6.1516	-0.4224
I	7:13	0.9958	1.0524	0.2787	-3.5877	8.0687	-0.6269

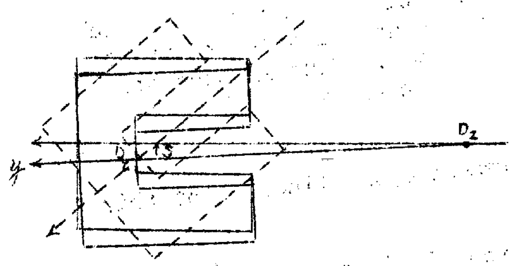


Figure 11
Form of the Vibration of
Type G (m:n = 7:7)
$P_x = 5.3530$ a, $P_z = -0.3211$ a

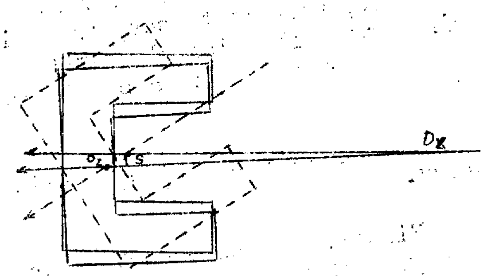


Figure 13
Form of the Vibration of
Type H (m:n = 7:9)
$P_x = 6.1516$ a, $P_z = -0.4224$ a

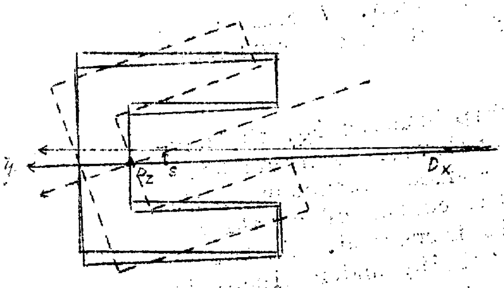


Figure 12
Form of the Vibration of
Type G (m:n = 9:9)
$P_x = 5.3508$ a, $P_z = -0.6314$ a

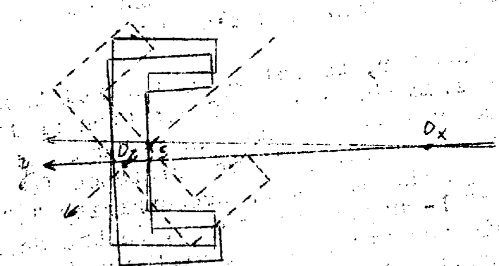


Figure 14
Form of the Vibration of
Type I (m:n = 7:13)
$P_x = 8.0687$ a, $P_z = -0.6269$ a

These findings, therefore, show that in ℶ -shaped structures, as in L-shaped structures, the lateral vibration distance from the center of rotation has a tendency to increase as the length of the sides of the plan figure decreases. Figures 11 to 14 show the form of vibration for the different types of structure.

6. Conclusions

In this thesis we have made a study of solid vibrations in one-story L- and ℶ -shaped frame structures in which the framework is distributed evenly and in which the rigidity of the frames is uniform longitudinally and laterally. In the case of frame structures not having symmetrical lateral and longitudinal axes, even though the mass is evenly distributed over the base area, the center of gravity does not coincide with the center of rigidity; that is to say, there is an eccentricity factor which causes the vibration along three dimensions. Thus in such structures there is rarely a purely lateral vibration or a torsional vibration.

By way of conclusion we will summarize the findings of this study.

Firstly, as regards the expression of vibration, there is a variation of barely 1% between lateral and rotational frequency-- $n_o = \sqrt{G/M}$ (that is, purely lateral vibration) whether in an L-shaped or in a ℶ -shaped structure.

Next, as regards characteristic functions, the forms of the plan figures are seen to have fairly marked differences, and comparing L- and ℶ -shaped structures, the latter has a larger degree of rotation in its lateral vibration than the former. Thus, it follows that ℶ -shaped structures have a greater tendency towards 3-dimensional vibration than L-shaped structures. A comparison between the distances from the center of rotation demonstrates this point even more clearly.

However, to follow up the relationship between P_x, the distance from the center of rotation of lateral vibration and the degree of eccentricity, in the case of ℶ -shaped structures, there is no great change in the distance from the center of rotation although the degree of eccentricity "e" is increased. In the case of L-shaped structures, in the eccentricity under discussion in this treatise, the P_x distance from the center of rotation will suddenly decrease because of the slight increase in the degree of eccentricity; thus three-dimensionality (or the rotational quality in lateral vibration) has a tendency to increase. This point must always be taken into consideration.

Taking the L-shaped types A to E and the ℶ -shaped types F to I comparing them one with another it will be seen that it is not the area of the floor space that determines the nature of the vibration, but the ratio between the length of the arms. That is to say, as the ratio between the length of the arms decreases

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